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A FRAMEWORK FOR PEAT BEHAVIOUR BASED ON HYPERPLASTICITY PRINCIPLES

D. Boumezerane[1], A. Makdisi[2], G. Grimstad[1]
[1]*Geotechnical Division, Norwegian University of Science and Technology, Trondheim, Norway,*
[2]*Department of Civil & Environmental Engineering, University of Washington, Seattle, WA, USA*

ABSTRACT: Presented here is the framework for a thermodynamically consistent model of the behavior of fibrous peat, using the principles of hyperplasticity. In hyperplasticity, the description of the behavior of a saturated material is based on the principal of mass conservation, the first and second laws of thermodynamics, and the existence of two scalar functions which essentially describe the stored internal energy and the rate of dissipation in the system. Depending on how the internal and dissipative energy functions are defined, one can derive the complete constitutive behavior of a material due to thermo-hydro-mechanical effects. The complex behavior of peat can be modelled using a poro-visco-elastoplastic model that accounts for such factors as stress-dependent skeleton stiffness, fiber compressibility, anisotropy, fluid flow in a porous medium, and rate-dependence. Differentiation of the energy functions allows for determination of the incremental stress-strain relationship and the evolution equations for the internal variable (in this case, the viscoplastic strain). The plastic multiplier, which is represented by a Lagrangian multiplier (Λ), can be solved for implicitly using the consistency model from Wang et al. (1997).

INTRODUCTION

Fibrous peat differs significantly from clay and sand in both its inherent properties and its resulting mechanical behavior. Peat generally consists of a network of long, hollow, flexible organic fibers in a loosely deposited, aqueous environment. Its very low initial density and extremely high natural water content result in viscous and highly compressible deformation behavior. Thus, creep deformations play a more significant role in settlement behavior of peat than for most other geomaterials. In the past, models used to describe the behavior of peat have been adapted from models originally developed to describe clay or sand, and thus may not be as inherently well-equipped to capture the complex deformation behavior of fibrous peat.

Houlsby & Puzrin (2006) developed an approach to modeling geomaterials using the principles of hyperplasticity, which involves the development of material models within a fully thermodynamically consistent framework. The formulation of a hyperplastic model must obey the principles of mass conservation, conservation of energy (First Law of thermodynamics), and the principle of work dissipation in the form of heat (Second Law of thermodynamics). The main advantage to employing this framework is that it cannot produce thermodynamically inconsistent results.

The formulation of a hyperplastic model requires the specification of two potential functions: one describing the internal energy of the system, from which

the constitutive behavior is defined; and one describing how energy is dissipated from the system, from which the evolution of the internal variables is determined. By defining these two functions such that the unique and complex properties of peat are accounted for in the internal energy and energy dissipation, a thermodynamically consistent model for peat can be formulated and implemented independently of the constraints of traditional soil models.

ENERGY FUNCTIONS

The relative importance of the flow of pore fluid on the deformation response of fibrous peat necessitates the development of a hyperplastic model within a poromechanical framework. For a porous continuum (Houlsby & Puzrin, 2006), the Gibbs' free energy function, which is a form of the internal energy function, is defined as:

$$g = g^s + wg^w \tag{1}$$

where g^s and g^w are the free energy functions for the solid and water phases, respectively, and w is the water content.

$$\begin{aligned} g^s = \; & g_0^s + (p-p_0)v_0^s - (\theta-\theta_0)s_0^s - \frac{(p-p_0)^2}{2K^s} + 3\alpha^s(\theta-\theta_0)(p-p_0)v_0^s \\ & - c_p^s\frac{(\theta-\theta_0)^2}{2\theta_0} \\ & - \frac{1}{\rho_0}\left[\frac{p_i^{2-n}}{p_r^{1-n}k(1-n)(2-n)} + \frac{\bar{\sigma}_{kk}}{3k(1-n)} + \alpha(\theta-\theta_0)\bar{\sigma}_{kk} + \bar{\sigma}_{ij}\alpha_{ij}\right] \end{aligned} \tag{2}$$

- g_0^s, v_0^s, and s_0^s are the initial free energy, specific volume, and specific entropy values for the solid phase, respectively
- K^s, α^s, and c_p^s are the isothermal bulk modulus, coefficient of thermal expansion, and mass heat capacity at constant pressure (p_0) of the solid particles. p_0, θ_0, and ρ_0 are the initial pore fluid pressure, temperature, and density of the total system, respectively. α is the linear thermal expansion coefficient of the skeleton matrix
- α_{ij} is the internal variable, in this case, the viscoplastic strain

The bulk and shear stiffness of the skeleton matrix, K and G, are considered to be pressure-dependent:

$$\frac{K}{p_r} = k\left[\frac{p}{p_r}\right]^n \quad \frac{G}{p_r} = g_s\left[\frac{p}{p_r}\right]^n \tag{3}$$

where p_r is the reference pressure, k and g_s are dimensionless proportionality constants, and the exponent n is between 0 and 1.

$$p_i^{2-n} = \left(\frac{\bar{\sigma}_{ii}\bar{\sigma}_{jj}}{9} + \frac{k(1-n)\bar{\sigma}'_{ij}\bar{\sigma}'_{ij}}{2g_s}\right)^{\frac{2-n}{2}} \tag{4}$$

The free energy of the water phase is:

$$g^w = g_0^w + (p-p_0)v_0^w - (\theta-\theta_0)s_0^w - \frac{(p-p_0)^2}{2K^s} + 3\alpha^w(\theta-\theta_0)(p-p_0)v_0^w - c_p^w\frac{(\theta-\theta_0)^2}{2\theta_0} \quad (5)$$

The second function required is the force potential function *z*, which is a form of the energy dissipation function:

$$z = \frac{(E+\psi_{ij}\bar{\sigma}_{ij})}{\rho_0}\sqrt{\tilde{\alpha}'_{ij}\tilde{\alpha}'_{ij}} + \frac{\Lambda}{\rho_0}\left(3\beta\sqrt{\tilde{\alpha}'_{ij}\tilde{\alpha}'_{ij}} + \tilde{\alpha}_{ii}\right) + \frac{v^w}{2\rho k_m}m_i m_i + \frac{\theta}{2\rho k_\eta}\eta_i\eta_i + \frac{k_2}{b}r^{1-b}\left[(\tilde{\alpha}_{ii}\tilde{\alpha}_{ii})^{b/2} + \left(\tilde{\alpha}'_{ij}\tilde{\alpha}'_{ij}\right)^{b/2}\right] \quad (6)$$

where $\tilde{\alpha}_{ij}$ and $\tilde{\alpha}'_{ij}$ are the total and deviatoric increments of the internal variable, taken here to be the plastic strain. The first term of the force potential accounts for energy dissipation due to interactions between peat fibers in the skeleton matrix, analogous to frictional dissipation in granular materials, where *E* is the tensile strength of the entire skeleton matrix. ψ_{ij} is introduced to account for fabric anisotropy, and modifies the dissipation term such that at low stresses, the dissipation due to fiber interactions is insignificant, and at high stress (near failure), the dissipation term converges to $\frac{E}{\rho_0}\sqrt{\tilde{\alpha}'_{ij}\tilde{\alpha}'_{ij}}$. The second term is related to the dissipation due to dilation or contraction of the skeleton matrix, where *β* is related to the dilation angle of the material, and Λ is the Lagrangian multiplier. The third and fourth terms describe the dissipation due to mass flux (due to pore water flow) and entropy flux (due to temperature changes) out of the system, respectively, where k_m is the permeability coefficient and k_η is the thermal conductivity coefficient. The fourth term accounts for the viscous behavior of the material, where $k_2 = \eta/r$, and η is the viscosity of the total material. *r* is a constant that maintains the stress dimensions of k_2 , and *b* is generally in the range of 1 to 2. The viscous terms describe the long-term flow of micropore fluid across the individual fiber membranes, which can occur as a result of both deviatoric stress and volumetric compression, and is principally responsible for the significant secondary compression in peat.

CONSTITUTIVE RELATIONSHIPS

The constitutive relationships can be derived, in a finite-strain regime, from the Gibbs' free energy expression as follows:

$$\Delta_{ij} = -\rho_0\frac{\partial g^s}{\partial\bar{\pi}_{ij}} \qquad \bar{\chi}_{ij} = -\rho_0\frac{\partial g^s}{\partial\alpha_{ij}} \quad (7)$$

$$v^s = \frac{\partial g^s}{\partial p^s}\ ,\ v^w = \frac{\partial g^w}{\partial p^w}\ ,\ s^s = -\frac{\partial g^s}{\partial\theta^s}\ ,\ s^w = -\frac{\partial g^w}{\partial\theta^w} \quad (8)$$

where Δ_{ij} is the Green-Lagrange strain tensor , π_{ij} is the 1st Piola-Kirchhoff stress tensor, v^s is the specific volume, and s^s is the entropy. The generalized and dissipative generalized stresses are defined by:

$$\bar{\chi}_{ij} = -\rho_0\frac{\partial g^s}{\partial\tilde{\alpha}_{ij}} \qquad \chi_{ij} = \rho_0\frac{\partial z^g}{\partial\tilde{\alpha}_{ij}} \quad (9)$$

By invoking Ziegler's condition of orthogonality, which requires the dissipated energy to be maximal (i.e. $\chi_{ij} = \bar{\chi}_{ij} = \bar{\sigma}_{ij}$), we obtain an expression for the deviatoric viscoplastic strain increment using the deviatoric form of Eq. 7:

$$\tilde{\alpha}'_{ij} = r\left[\frac{\langle\sqrt{\bar{\sigma}'_{kl}\bar{\sigma}'_{kl}} - (E + \psi_{ij}\bar{\sigma}_{ij} + 3\Lambda\beta)\rangle}{k_2}\right]^{\frac{1}{b-1}} S_{ij}(\bar{\sigma}_{ij}), \qquad S(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x < 0 \end{cases} \tag{10}$$

The Lagrangian multiplier, Λ, must then be solved for using the volumetric form of Eq. 7: $3\Lambda = \chi_{ii} - k_2 r^{1-b}\tilde{\alpha}_{ii}[\tilde{\alpha}_{ii}\tilde{\alpha}_{ii}]^{\frac{b}{2}-1}$ (11)

The implicit expression for Λ can be solved within the finite-element framework using an implicit stress-update algorithm (Wang 1997), based on the consistency condition of the yield surface for a viscoplastic model:

$$\tilde{y} = \frac{\partial y}{\partial \chi_{ij}}\tilde{\chi}_{ij} + \frac{\partial y}{\partial \alpha_{ij}}\tilde{\alpha}_{ij} + \frac{\partial y}{\partial \psi_{ij}}\tilde{\psi}_{ij} + \frac{\partial y}{\partial \tilde{\alpha}_{ij}}\tilde{\tilde{\alpha}}_{ij} = 0 \tag{12}$$

where *y* can be obtained directly from the force potential *z* through the transformation:

$$y = \bar{\chi}_{ij}\tilde{\alpha}_{ij}/\rho_0 - z\ , \qquad \bar{\chi}_{ij} = \bar{\sigma}_{ij} \tag{13}$$

CONCLUSION

A framework based on hyperplasticity is proposed for rate-dependent peat behavior. This thermo-poro-viscoplastic formulation accounts for fabric anisotropy, volumetric and deviatoric viscosity, and large deformations governed by pore fluid flow. The formulation is based on internal and dissipated energy function, and the constitutive relations are derived using Gibbs free energy and force potential (dissipation) functions. The model, which is yet to be implemented, shows promise as a thermodynamically consistent model developed specifically for peat-like materials.

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